7. $\int \frac{x^3+5x^2+4x+1}{x^2}\,dx$

$$I = \int \frac{x^3+5x^2+4x+1}{x^2}\,dx$$

$$= \int \frac{x^3}{x^2}\,dx + 5\int \frac{x^2}{x^2}\,dx + 4\int \frac{x}{x^2}\,dx + \int \frac{1}{x^2}\,dx$$

$$= \int x\,dx + 5\int dx + 4\int \frac{1}{x}\,dx + \int x^{-2}\,dx$$

$$= \frac{x^2}{2} + 5x + 4\log x + \frac{x^{-2+1}}{-2+1} + c$$

$$= \frac{x^2}{2} + 5x + 4\log x - \frac{1}{x} + c$$

ii) $\int \frac{x^3+3x+4}{\sqrt{x}}$

$$I = \int \frac{x^3+3x+4}{\sqrt{x}}\,dx$$

$$= \int \frac{x^3}{\sqrt{x}}\,dx + 3\int \frac{x}{\sqrt{x}}\,dx + \int \frac{4}{\sqrt{x}}\,dx$$

$$= \int x^3 \cdot x^{-1/2}\,dx + 3\int x^1 \cdot x^{-1/2} + 4\int x^{-1/2}\,dx$$

$$= \int x^{3-\frac{1}{2}}\,dx + 3\int x^{1/2}\,dx + 4\int x^{-1/2}\,dx$$

$$= \int x^{5/2}\,dx + 3\int x^{1/2}\,dx + 4\int x^{-1/2}\,dx$$

$$= \frac{x^{5/2+1}}{\frac{5}{2}+1} + 3\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + 4\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c$$

$$= \frac{x^{7/2}}{\frac{7}{2}} + 3\frac{x^{3/2}}{\frac{3}{2}} + 4\frac{x^{1/2}}{\frac{1}{2}} + c$$

Ans

ii) $\int \frac{x^3-x^2+x-1}{x-1} dx$

$$I = \int \frac{x^3-x^2+x-1}{x-1} dx$$

$$= \int \frac{x^2(x-1)+1(x-1)}{x-1} dx$$

$$= \int \frac{(x-1)(x^2+1)}{x-1} dx$$

$$\int (x^2+1) dx$$

$$= \int x^2 dx + \int 1 \cdot dx + c$$

$$I = \frac{x^3}{3} + x + c$$

10.i) $\int \frac{1-\sin x}{\cos^2 x}\, dx$

$$I = \int \frac{1-\sin x}{\cos^2 x}\, dx$$

$$= \int \frac{1}{\cos^2 x}\, dx - \int \frac{\sin x}{\cos^2 x}\, dx$$

$$= \int \sec^2 x\, dx - \int \tan x \sec x\, dx$$

$$\boxed{I = \tan x - \sec x + c}$$

ii) $\int \frac{e^{6\log x} - e^{5\log x}}{e^{4\log x} - e^{3\log x}}\,dx$

$$I = \int \frac{e^{6\log x} - e^{5\log x}}{e^{4\log x} - e^{3\log x}}\,dx$$

$$I = \int \frac{e^{\log x^6} - e^{\log x^5}}{e^{\log x^4} - e^{\log x^3}}\,dx$$

$$I = \int \frac{x^6 - x^5}{x^4 - x^3}\,dx$$

$$\int \frac{x^5(x-1)}{x^3(x-1)}\,dx$$

$$= \int x^2\,dx$$

$$\boxed{I = \frac{x^3}{3} + c}$$

11 i) $\int \sec x\,(\sec x + \tan x)\,dx$

$$I = \int \sec x\,(\sec x + \tan x)\,dx$$

$$= \int \sec^2 x\,dx + \int \sec x \tan x\,dx$$

$$\boxed{I = \tan x + \sec x + c}$$

ii) $\int \frac{sec^2x}{cosec^2x}dx$

$I = \int \frac{Sec^2x}{Cosec^2x}dx$

$= \int \frac{\frac{1}{Cos^2x}}{\frac{1}{Sin^2x}}dx$

$= \int \frac{1}{Cos^2x} \times Sin^2x \, dx$

$\int \frac{Sin^2x}{Cos^2x}dx$

$Sec^2x = 1 + tan^2x$

$\int tan^2x \, dx$

$\int (Sec^2x - 1)dx$

$= \int Sec^2x \, dx - \int 1 \cdot dx$

$I = tanx - x + C$

12 i) $\int \left(\frac{cotx}{sinx} - \frac{tanx}{cosx} - 3tan^2x\right) dx$

$$I = \int (\text{Cot}x\ \text{Cosec}x - \tan x\ \text{Sec}x - 3\tan^2 x)\, dx$$

$$= \int \text{Cot}x\ \text{Cosec}x\, dx - \int \tan x\ \text{Sec}x\, dx - \int 3(\text{Sec}^2 x - 1)\, dx$$

$$= -\text{Cosec}x - \text{Sec}x - \left[3\int \text{Sec}^2 x\, dx - 3\int 1 \cdot dx\right]$$

$$\boxed{I = -\text{Cosec}x - \text{Sec}x - 3\tan x + 3x + c}$$